Integrace substituční metodou

Pro každou primitivní funkci určete její definiční obor !

Vypočítejte:

a) $\int (2x+6)^2 \mathrm{d}x$, $\int (x+1)^{21} \mathrm{d}x$;

b) $\int \sqrt{5x+2}\, \mathrm{d}x$, $\int \sqrt[10]{2x-3}\, \mathrm{d}x$;

c) $\int (ax+b)^n \mathrm{d}x$, $\int \sqrt[n]{ax+b}\, \mathrm{d}x$ ($a,b \in \mathbb{R}$, $a \neq 0$, $n \in \mathbb{N}$);

d) $\int \sin(-x+1)\, \mathrm{d}x$, $\int \cos(2x)\, \mathrm{d}x$;

e) $\int \sin(ax+b)\, \mathrm{d}x$, $\int e^{ax+b}\, \mathrm{d}x$ ($a,b \in \mathbb{R}$, $a \neq 0$, $n \in \mathbb{N}$).

Výsledky:

a) $\left[\frac{1}{6}(2x+6)^3 + C\right] \left[\frac{1}{22}(x+1)^{22} + C\right]$;

b) $\left[\frac{2}{15}\sqrt{(5x+2)^3} + C\right] \left[\frac{5}{11}\sqrt[10]{(2x-3)^{11}} + C\right]$;

c) $\left[\frac{1}{a(n+1)}(ax+b)^{n+1} + C\right] \left[\frac{n}{a(n+1)}\sqrt[n]{(ax+b)^{n+1}} + C\right]$;

d) $\left[\cos(x-1) + C\right] \left[\frac{1}{2}\sin(2x) + C\right]$;

e) $\left[-\frac{1}{a}\cos(ax+b) + C\right] \left[\frac{1}{a}e^{ax+b} + C\right]$.

Vypočítejte:

a) $\int x(x^2+6)^{19} \mathrm{d}x$, $\int x(x^2+6)^n \mathrm{d}x$ ($n \in \mathbb{N}$);

b) $\int x\sqrt{x^2+1}\, \mathrm{d}x$, $\int \frac{x}{\sqrt{x^2+1}} \mathrm{d}x$, $\int \frac{x}{(x^2+1)\sqrt{x^2+1}} \mathrm{d}x$;

c) $\int \frac{x}{x^2+4} \mathrm{d}x$, $\int \frac{x^2}{x^3-8} \mathrm{d}x$, $\int \frac{x^{n-1}}{x^n - a^n} \mathrm{d}x$ ($a \in \mathbb{R}$, $a \neq 0$, $n \in \mathbb{N}$).

Výsledky:

a) $\left[\frac{1}{40}(x^2+6)^{20} + C\right] \left[\frac{1}{2n+2}(x^2+6)^{n+1} + C\right]$;

b) $\left[\frac{1}{3}\sqrt{(x^2+1)^3} + C\right] \left[\sqrt{x^2+1} + C\right] \left[-1/\sqrt{x^2+1} + C\right]$;

c) $\left[\frac{1}{2}\ln(x^2+4) + C\right] \left[\frac{1}{3}\ln\left|x^3-8\right| + C\right] \left[\frac{1}{n}\ln\left|x^n - a^n\right| + C\right]$.

Vypočítejte:

a) $\int \frac{1}{4+x^2} \mathrm{d}x$, $\int \frac{1}{a^2+x^2} \mathrm{d}x$ ($a > 0$);

b) $\int \frac{1}{\sqrt{1-9x^2}} \mathrm{d}x$, $\int \frac{1}{\sqrt{1-ax^2}} \mathrm{d}x$ ($a > 0$);

c) $\int \frac{e^x}{1+e^{2x}} \mathrm{d}x$, $\int \frac{e^x}{\sqrt{1-e^{2x}}} \mathrm{d}x$.

Výsledky:

a) $\left[\frac{1}{2}\operatorname{arctg}\frac{x}{2} + C\right] \left[\frac{1}{a}\operatorname{arctg}\frac{x}{a} + C\right]$;

b) $\left[\frac{1}{3}\arcsin(3x)+C\right]\left[\frac{1}{\sqrt{a}}\arcsin\left(x\sqrt{a}\right)+C\right]$;

c) $\left[\operatorname{arctg} e^x + C\right]\left[\arcsin e^x + C\right]$.

Vypočítejte:

a) $\int \sin^2 t \cos t \, dt$, $\int \sin^n t \cos t \, dt$ ($n \in \mathbb{N}$);

b) $\int \cos^2 t \sin t \, dt$, $\int \cos^n t \sin t \, dt$ ($n \in \mathbb{N}$);

c) $\int \sin^4 t \cos^3 t \, dt$;

d) $\int \frac{\cos t}{\sin^4 t} dt$;

e) $\int \frac{\sin^3 t}{\cos^7 t} dt$;

f) $\int \sin^5 t \, dt$, $\int \cos^5 t \, dt$;

g) $\int \operatorname{tg} t \, dt$, $\int \operatorname{cotg} t \, dt$;

h) $\int \frac{\cos x - \sin x}{\sqrt{\cos x + \sin x}} dx$. **Návod**: Volte substituci $u = \cos x + \sin x$.

Výsledky:

a) $\left[\frac{1}{3}\sin^3 t + C\right]\left[\frac{1}{n+1}\sin^{n+1} t + C\right]$;

b) $\left[-\frac{1}{3}\cos^3 t + C\right]\left[-\frac{1}{n+1}\cos^{n+1} t + C\right]$;

c) $\left[\frac{1}{5}\sin^5 t - \frac{1}{7}\sin^7 t + C\right]$;

d) $\left[-\frac{1}{3}\sin^{-3} t + C\right]$;

e) $\left[\frac{1}{6}\cos^{-6} t - \frac{1}{4}\cos^{-4} t + C\right]$;

f) $\left[-\cos t + \frac{2}{3}\cos^3 t - \frac{1}{5}\cos^5 t + C\right]\left[\sin t - \frac{2}{3}\sin^3 t + \frac{1}{5}\sin^5 t + C\right]$;

g) $\left[-\ln|\cos t| + C\right]\left[\ln|\sin t| + C\right]$;

h) $\left[2\sqrt{\cos x + \sin x} + C\right]$.

Vypočítejte:

a) $\int \sqrt{f(x)}\, f'(x) \, dx$, $\int \frac{f'(x)}{\sqrt{f(x)}} dx$;

b) $\int f^n(x) f'(x) \, dx$;

c) $\int \frac{1}{1+f^2(x)} f'(x) \, dx$;

d) $\int \frac{f(x)}{1+f^2(x)} f'(x) \, dx$;

e) $\int f'(x) f''(x) \, dx$, $\int f^{(n)}(x) f^{(n+1)}(x) \, dx$.

Výsledky:

a) $\left[\frac{2}{3}\sqrt{f^3(x)} + C\right]\left[2\sqrt{f(x)} + C\right]$;

b) $\left[\frac{1}{n+1} f^{n+1}(x) + C \text{ pro } n \neq -1,\ \ln|f(x)| \text{ pro } n = -1\right]$;

c) $\left[\operatorname{arctg}(f(x)) + C\right]$;

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d) $\left[\frac{1}{2}\ln(1+f^2(x))+C\right]$;

e) $\left[\frac{1}{2}\left(f'(x)\right)^2+C\right]\left[\frac{1}{2}\left(f^{(n)}(x)\right)^2+C\right]$.
